



Free Material Optimization of Composite Structures

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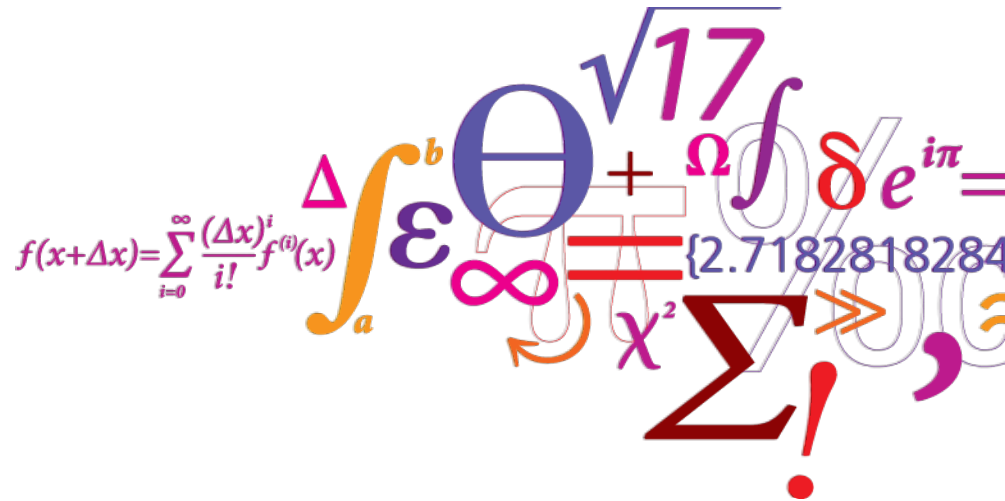
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Free Material Optimization of Composite Structures

Alemseged G Weldeyesus, PhD student

Wind Turbines section



Structural optimization

- Find the lightest structure that is able to carry a given set of loads.
 minimize weight
 s.t. compliance \leq given value
- Find the stiffest structure that is able to carry a given set of loads with limited amount of material.
 minimize compliance
 s.t. weight \leq given value

Structural optimization

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In Free Material Optimization (FMO)

The design variable is the full material tensor which can vary almost freely at each point of the design domain.

Structural optimization

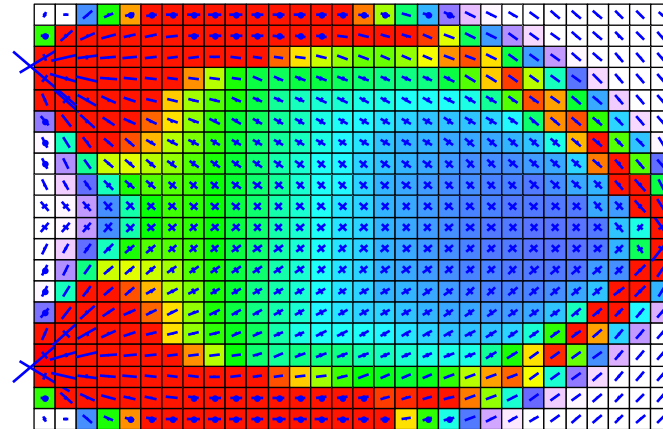
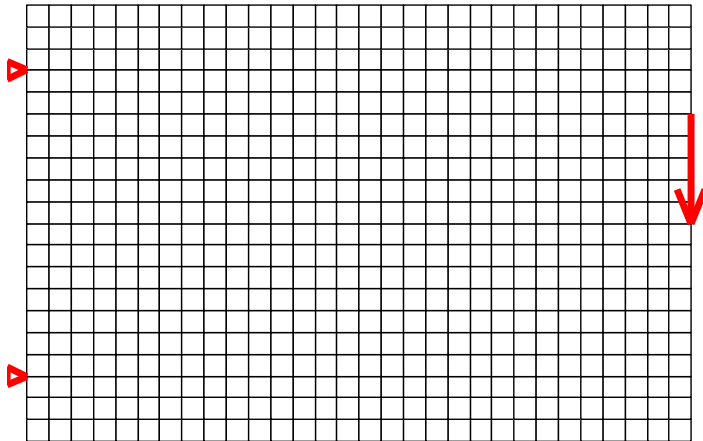
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In Free Material Optimization (FMO)

The design variable is the full material tensor which can vary almost freely at each point of the design domain.

FMO yields optimal distribution of material as well as optimal local material properties.

A 2D example



Design Domain, bc and force

Optimal density distribution

The obtained design

- can be considered as an ultimately best structure,
- is difficult and expensive to manufacture,
- can be used to generate benchmarks and to propose novel ideas for new design situations.

FMO formulations

Mechanical assumptions

- static loads
- linear elasticity

FMO formulations

Mechanical assumptions

- static loads
- linear elasticity

Basic FMO formulations for solid structures

Minimum compliance problem

$$\begin{aligned} \inf_{u \in V, E \in \tilde{\mathcal{E}}} \quad & \sum_{l \in L} w_l f_l^T u_l \\ \text{Subject to} \quad & A(E)u_l = f_l, \quad l \in L, \\ & \sum_{i=1}^m \text{Tr}(E_i) \leq \bar{V} \\ & \tilde{\mathcal{E}} = \{E \in (S) \mid \underline{\rho} \leq \text{Tr}(E_i) \leq \bar{\rho}, E \succeq 0\} \end{aligned}$$

Minimum weight problem

$$\begin{aligned} \inf_{u \in V, E \in \tilde{\mathcal{E}}} \quad & \sum_{i=1}^m \text{Tr}(E_i) \\ \text{Subject to} \quad & A(E)u_l = f_l, \quad l \in L, \\ & \sum_{l \in L} w_l f_l^T u_l \leq \bar{\gamma} \end{aligned}$$

Other different formulations can also be derived.

Additional constraints

Depending on the problem formulation additional constraints can be included such as constraints

- on local stresses

$$\sum_{k=1}^G \|E_i B_{ik} u\|^2 \leq s_\sigma$$

- on local strains

$$\sum_{k=1}^G \|B_{ik} u\|^2 \leq s_\sigma$$

- on displacement

$$Cu \leq d$$

- etc

Optimization method

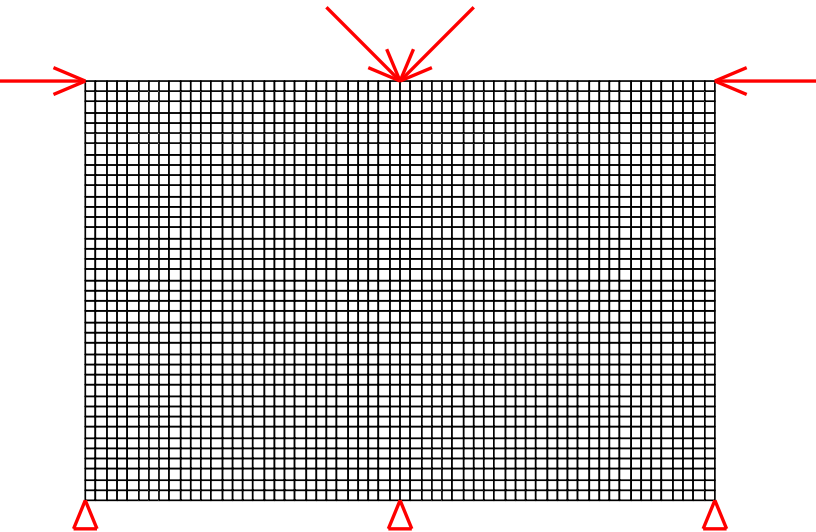
The resulting optimization problem is a nonlinear semidefinite program, a non-standard problem **with many matrix inequalities**.

Existing robust and efficient primal-dual interior point methods for nonlinear programming has in this project been extended to solve FMO problems.

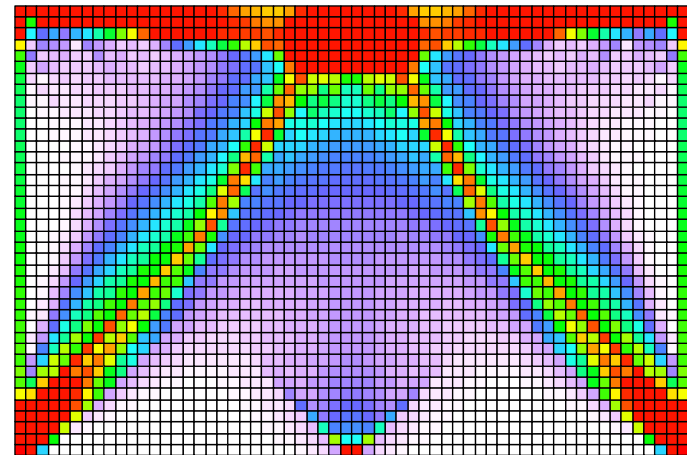
Numerical Results

Design domain, bc and loads

4-loads



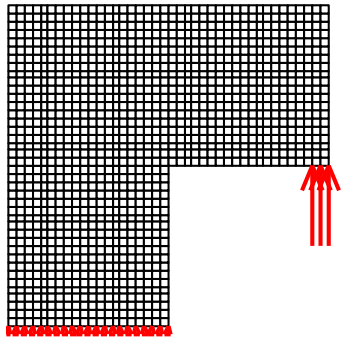
Optimal material density distribution



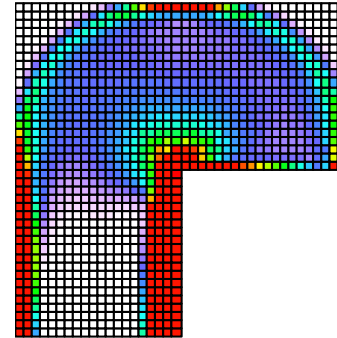
Numerical Results

With out stress constraints

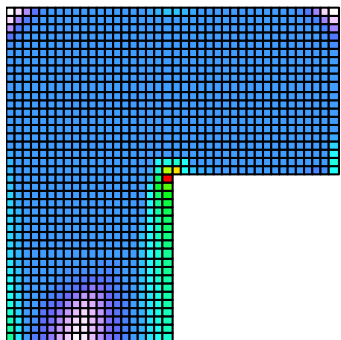
Design Domain, bc and forces



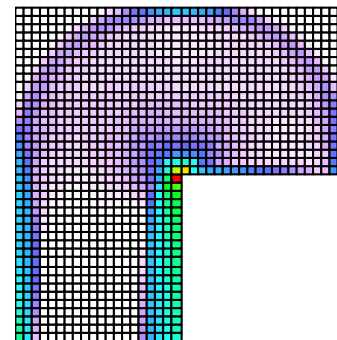
Density distribution of optimal design



Optimal strain norms



Optimal stress norms

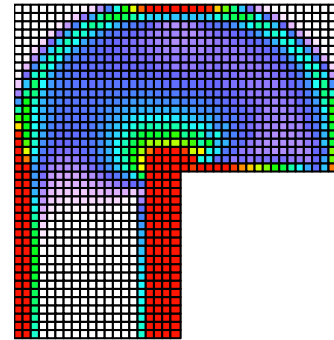


Numerical Results

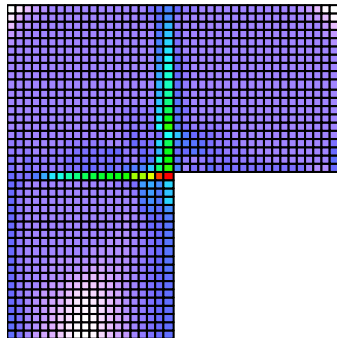
With stress constraints,

- max stress is decreased by 50%
- 11 more iterations

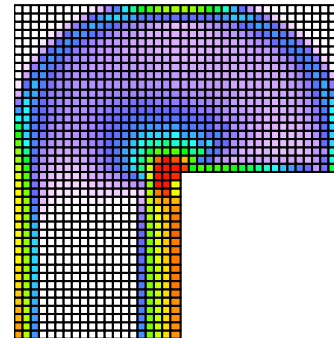
Density distribution of optimal design



Optimal strain norms



Optimal stress norms



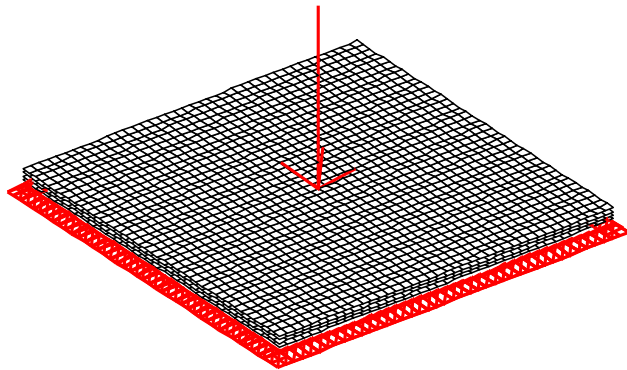
- Higher stresses are distributed to the neighbor regions,
- So is also material distribution

Numerical Results

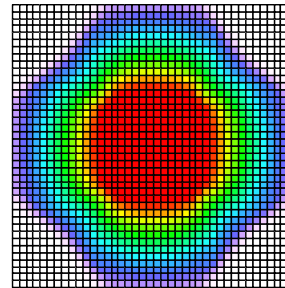
Design domain, bc and loads

optimal material density distribution

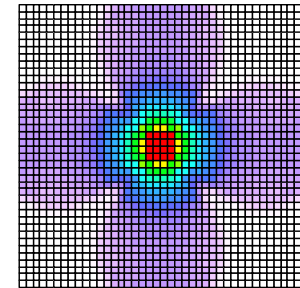
4-layers



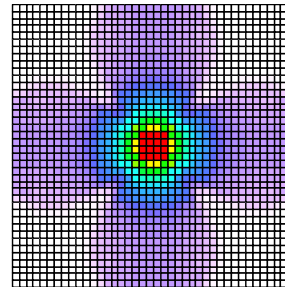
Density distribution, Layer=1



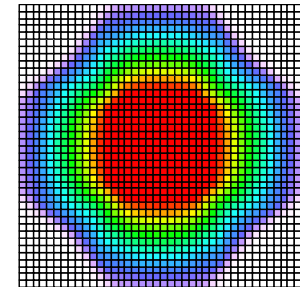
Density distribution, Layer=2



Density distribution, Layer=3



Density distribution, Layer=4



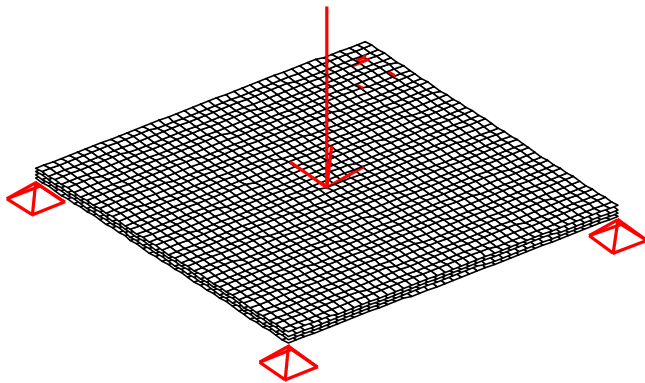
- Symmetric laminate
- High material distribution on top and bottom layers

Numerical Results

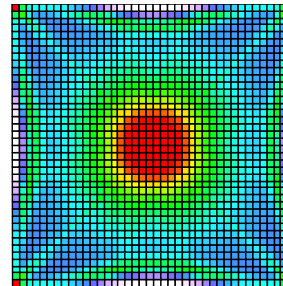
Design domain, bc and loads

optimal material density distribution

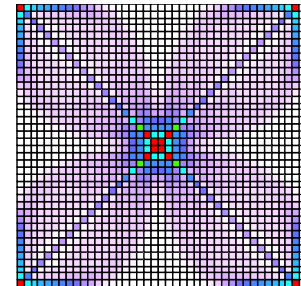
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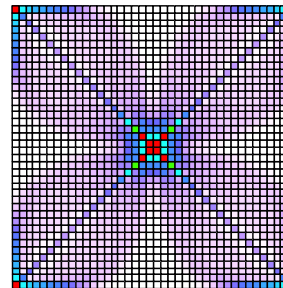
Density distribution, Layer=1



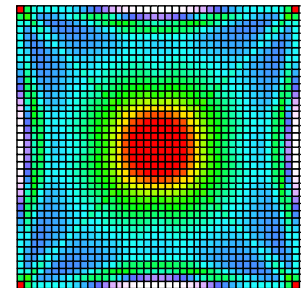
Density distribution, Layer=2



Density distribution, Layer=3



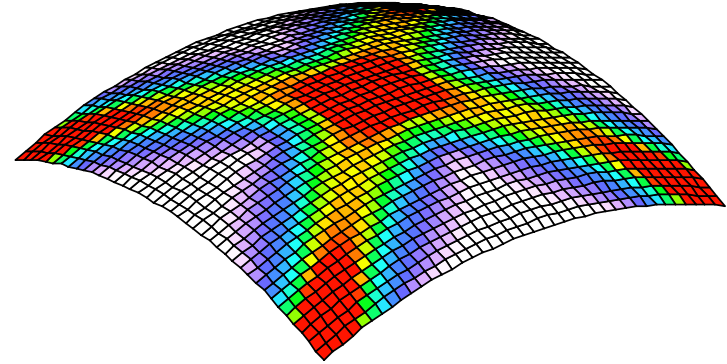
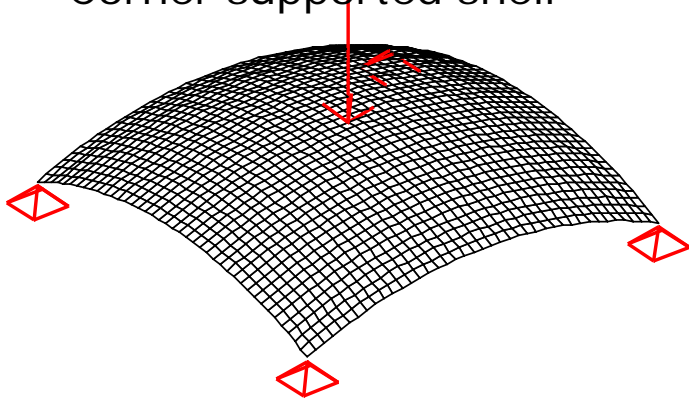
Density distribution, Layer=4



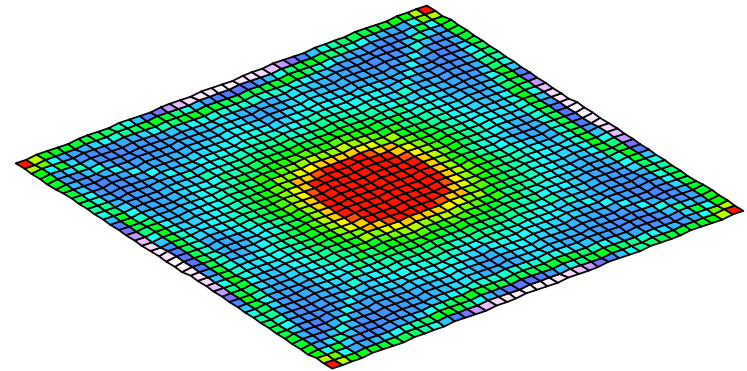
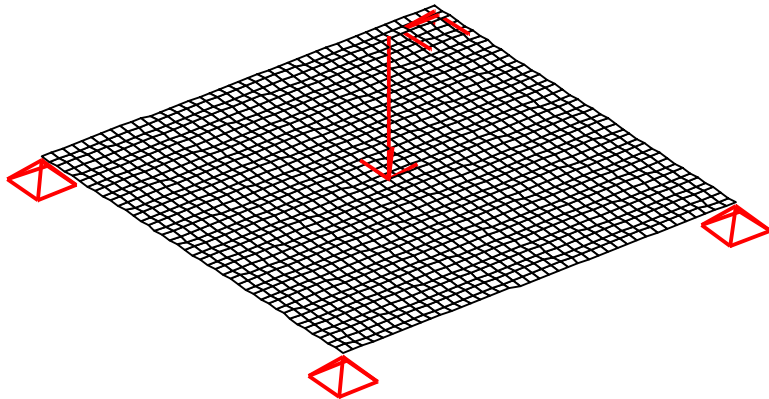
- Symmetric laminate
- High material distribution on top and bottom layers

Numerical Results

Corner supported shell

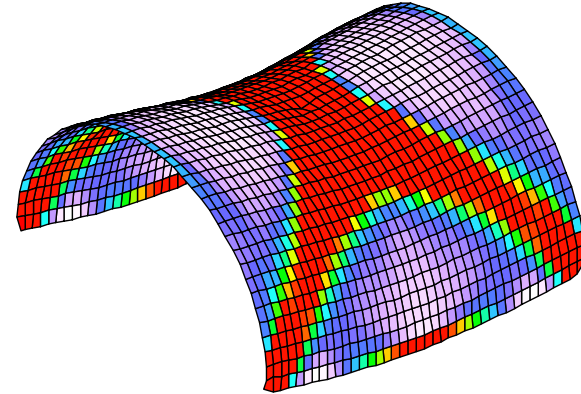
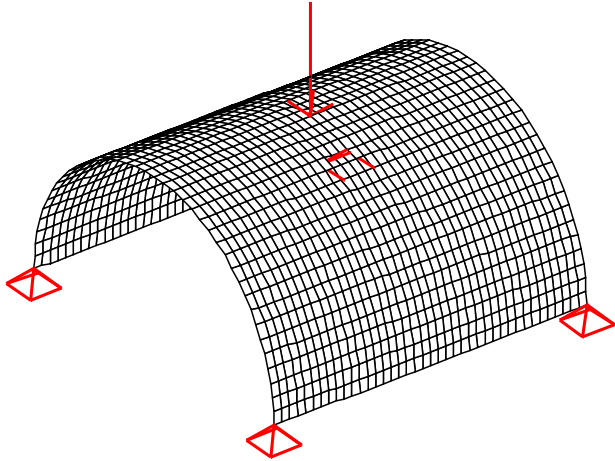


Corner supported plate

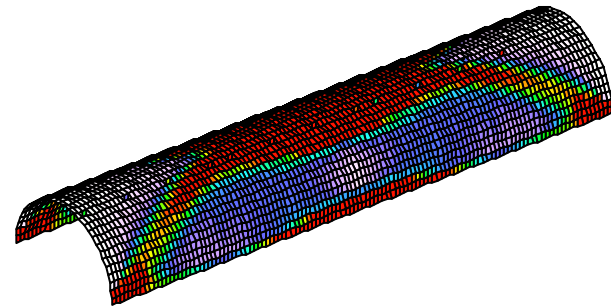
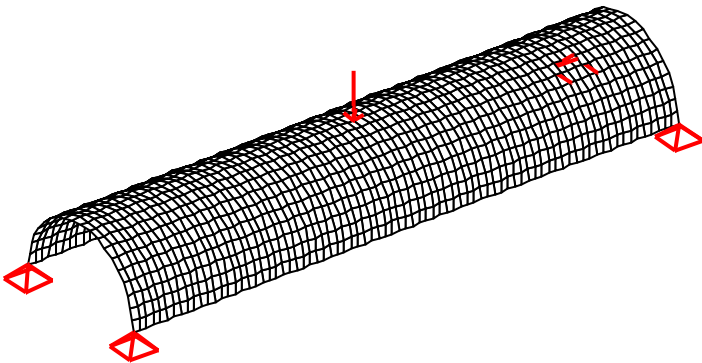


Numerical Results

Corner supported half cylinder, length to width ratio 1:1

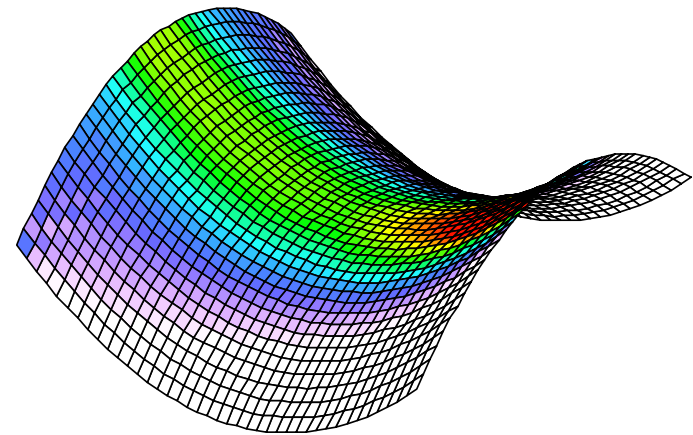
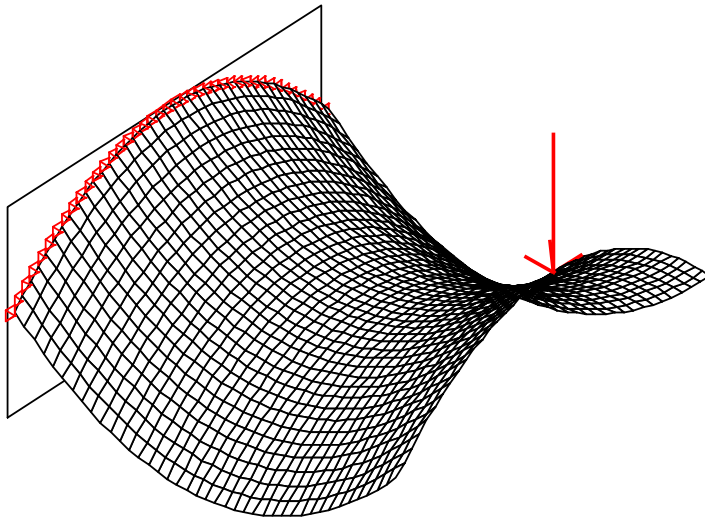


Corner supported half cylinder, length to width ratio 4:1



Numerical Results

Edge supported saddle surface shell



Future work

- Provide physical interpretation of the optimal solution
- Propose models and methods for FMO for beams with advanced cross sectional analysis (with José Blasques and Mathias Stolpe at DTU Wind Energy)
- Include large deformations (geometric nonlinearity)
- Write articles, and PhD thesis.